MECHANICALLY AIDED HEAT TRANSFER

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NOMENCLATURE

A	Heat transfer surface at inside diameter of shell, ft^2
a	Blade factor, dimensionless. See Eq. (26)
A', a'	Constants in viscosity equations, dimensionless
b	Blade thickness, ft
B', b'	Constants in viscosity equations, dimensionless
D	Linear dimension in the Reynolds number, ft
F_c	Centrifugal force on blade, lb
F_d	Drag force on blade, lb
F_l	Lift force on blade, lb
f_c	Centrifugal force per unit volume of fluid, lb/ft^3
f_g	Gravitational force per unit volume of fluid, lb/ft^3
g	Gravitational constant, 4.18×10^8, ft/hr^2
h	Heat transfer coefficient of process fluid, Btu/hr x ft^2 x °F
K_1	Numerical factor, dimensionless
k	Thermal conductivity, Btu/hr x ft^2 x °F/ft
L	Length of blade, ft
m	Mass of blade, lb
n	Number of blades on rotor, dimensionless
n'	Flow behavior index, dimensionless
P_d	Power to overcome drag resistance, ft x lb/hr
P_s	Power to accelerate fraction s of holdup, ft x lb/hr
Q	Heat transferred, Btu/hr

R	Radius of shell, ft
Re	Reynolds number, dimensionless
r	Radial co-ordinate
s	Fraction of holdup accelerated to blade velocity, dimensionless
T	Temperature in viscosity equation, °R
Δt_w	Temperature difference between shell inside wall and vapor, °F
u	Linear velocity of blade, ft/hr
V	Downward flow, ft^3/hr
V_f	Downward flow in fillet, ft^3/hr
V_o	Feed, ft^3/hr
V_w	Downward flow in wall film, ft^3/hr
W	Holdup volume, ft^3
w	Width of flat blade face, ft
X	Cross-sectional area of fillet, ft^2
x, y, z	Cartesian co-ordinates
$\dot{z}$	Linear downflow velocity, ft/hr
α	Fraction of feed volume vaporized in processor, dimensionless
α', β'	Constants in viscosity equations, dimensionless
γ	Rate of shear, 1/hr
δ	Clearance between blade tip and shell, ft
δ_o	Clearance at mid-point of flat blade face and the shell, ft
ξ	Angle between resultant force on liquid and the horizontal, °
θ	Angle between flat blade face and the shell, °

λ	Latent heat of fluid, Btu/lb
μ	Viscosity, lb/ft x hr
μ_1	Viscosity at unit rate of shear, lb/ft x hr
μ_o	Viscosity of solvent, lb/ft x hr
μ_r	Reduced viscosity, lb/ft x hr
ρ	Density, lb/ft^3
τ	Shear stress, lb/ft^2
ϕ	Concentration, volume fraction
ϕ_o	Concentration of feed, volume fraction
ψ	A function
ω	Angular velocity of blade, radians/hr

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by

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ABSTRACT

Heat transfer devices with moving parts implement phase changes to and from viscous Newtonian and non-Newtonian fluids. By combining certain principles of heat and mass transfer, hydrodynamics and rheology, equations are developed for the design of these machines and the prediction of their performance. A numerical example illustrates the use of the derived equations.

MECHANICALLY-AIDED HEAT TRANSFER

INTRODUCTION

Changes of phase involving very viscous or foamy materials have frequently posed a serious problem in chemical engineering technology. Resulting from the growth of high vacuum processing and the field of polymer chemistry in general, the vaporization of a volatile component must frequently take place from a viscous fluid phase.

In a majority of operations the heat supplied for vaporization must be introduced into the fluid phase by convection or conduction. There is a practical limit of viscosity under which this heat can be introduced in conventional types of tubular equipment. Because of the desirability of avoiding streamline flow in shell and tube equipment the limiting viscosity is roughly from 50 to 100 centipoises.

With many polymeric solutions, with suspensions, slurries and pastes, viscosities of 100,000 centipoises are not unusual. Because these materials also may have high molecular weights and become unstable at elevated temperatures, it is necessary to effect the phase change very rapidly. In addition, the problem may also involve scale formation in the residual material, residue adhesion at the heat transfer surface, or nucleate bodying tendencies in the material being processed.

During the past decade several commercial devices have appeared. In these the heat transfer accompanying the phase change is mechanically

aided by the use of moving mechanical elements instead of hydrostatic head as in shell and tube equipment. A fortuitous condition lies in the fact that many of the very viscous fluids exhibit pseudoplastic non-Newtonian viscosity characteristics. In these cases the viscosity of the fluid may be greatly reduced at a given temperature by virtue of the movement of the mechanical element. The reader desiring greater familiarity with non-Newtonian characteristics is referred to the monograph of Metzner[1] which was written particularly for the chemical engineer.

These devices, which we shall refer to as mechanically-aided thermal processors, are used for such operations as concentration by vaporization, for absorption, stripping, deodorization, reaction, evaporation, and sensible heat transfer. The feed may contain aqueous or organic solvents, solutes of high or low molecular weight, or even suspended solids. Establishing the optimum values for dimensions, rotor speed, jacket temperature and throughput to obtain a desired process change is a complex but important problem with feed liquids having different thermal, chemical and rheological properties.

The commercial and periodical literature contain no direct references for the design of thermal processors. This paper presents a design analysis combining the theoretical principles of heat and mass transfer, hydrodynamics and rheology. Particular relationships are established involving the heat transfer coefficient, power requirement, throughput, and holdup as functions of processor geometry and operation and the fluid properties.

THE ROLE OF VISCOSITY

The liquids processed in mechanically-aided thermal processors are usually very viscous. Such liquids commonly show large changes in viscosity with comparatively small changes in concentration or temperature. In addition, they are usually non-Newtonian and it is not uncommon for their viscosities to decrease by a factor of 100 as the shear rate increases from a low value to a very high value. When it is realized that viscosity determines holdup and power consumption, it becomes apparent that this property of the fluid plays a vital role in the operation of such machines.

The viscosity of a Newtonian or non-Newtonian fluid is defined as the ratio of the shear stress τ to the rate of shear γ.

$$\mu = \tau/\gamma \tag{1}$$

Variation of μ with rate of shear can frequently be expressed by an exponential equation first proposed by Porter and Rao[2], and recently applied to engineering calculations by Metzner[3].

$$\mu = \mu_1 \gamma^{n'-1} \tag{2}$$

Here μ_1 is the viscosity at unit rate of shear and n' is a flow-behavior index being unity for Newtonian fluids and less than unity for most non-Newtonians.

The shear rate effective in a typical mechanically-aided processor can be estimated by dividing the linear blade velocity u by the clearance δ.

$$\gamma = u/\delta \tag{3}$$

With a blade velocity of 40 ft per sec and a clearance of 1/32 inch, this

gives a shear rate of 15,000 radians per second. The shear rates prevalent in typical laboratory viscometers are of the order of 100 radians per second. The laboratory measurement of the viscosity of a non-Newtonian material must be performed at several known rates of shear so that μ_1 and n' may be found from Eq. (2), thus permitting the viscosity to be computed at the operational shear rate of the processor.

Viscosity-concentration data can frequently be correlated by empirical relationships such as

$$\mu_r = \exp\left(\frac{a'\phi}{1 - b'\phi}\right) \tag{4}$$

or

$$\mu_r = \left(\frac{1 - \alpha'\phi}{1 - \beta'\phi}\right)^2 \tag{5}$$

In these equations μ_r is the "reduced viscosity", i.e., the ratio of the viscosity of the solution or suspension to that of the pure solvent or suspending medium, and ϕ is the volume fraction of the solute or disperse phase. The parameters a' and b' of Eq. (4) or α' and β' of Eq. (5) are fitted to data on the system. Frequently it is found that μ_r is temperature-independent. Since solvent viscosities follow the relation

$$\mu_o = A'e^{B'/T} \tag{6}$$

it is thus possible to estimate viscosities at one temperature from data obtained at another if the characteristic constants A' and B' of the solvent are known.

ROTORS WITH FIXED BLADES

An example of the mechanically-aided thermal processor is shown in Fig. 1 and consists of a vertical cylindrical shell fitted with a coaxial internal rotor and an external jacket for introducing or removing heat. Above the jacketed portion of the shell a mechanical entrainment separator operates from the same shaft. The blades of the rotor are made of metal or plastic plates secured firmly to the shaft radially or tangentially. The clearance between the blade tips and the shell wall is kept as small as possible while accommodating the machined tolerances of the rotor and the shell. The feed descends by gravity along the inner wall of the shell while the blades of the rotor spread the fluid over it. With high rotational speeds and close tolerances the fluid is vigorously agitated and the exposed surface of the fluid is continuously renewed, thereby implementing both heat and mass transfer.

The action of the processor assembly is illustrated in Fig. 2 which is a schematic drawing through the thermal section. A fillet of liquid is driven around the periphery of the rotor by the rotor blade. This fillet exchanges heat and mass with the fluid adhering to the shell and prevents the accumulation of a liquid film resistant to heat and mass transfer.

The limiting resistance to heat transfer within the shell is the wall film itself. The thickness of the film is taken for mathematical analysis to be equal to the clearance between the blade tip and the wall. The free surface of the fillet is kept in turbulence, facilitating mass transfer between

the liquid and vapor phases.

Consider a processor whose inner shell surface has a radius R and length L. It is fitted with an n-bladed coaxial rotor; the blades of thickness b have parallel sides which radiate from the axis, establishing a clearance δ between the blade tips and the cylinder wall. The rotor is driven at an angular velocity of ω radians per second. A fluid of density ρ and viscosity μ is fed into the top of the unit at the rate of V volume units per unit time.

Even without introducing the factors of heat and mass transfer, we are faced with an interesting hydrodynamic problem in attempting to compute the manner in which the fluid will descend through the processor. The descent can take place along four paths: (1) by drainage along the surfaces of the rotor blades; (2) by drainage along the cylinder wall; (3) by downflow in the fillet of fluid which will be carried in front of the blade tips; and (4) by the descent of droplets or fluid masses through the free volume between the shaft and the shell. If the rotors are moving with any workable velocity, it will be shown that (1) and (4) will not make an appreciable contribution. For fluid which is not acted upon by the wall, the resultant of the centrifugal force f_c and the gravitational force f_g will make an angle ζ with the horizontal

$$\zeta = \tan^{-1} f_g/f_c \qquad (7)$$

The respective forces per unit volume of fluid are

$$f_g = \rho g \quad (8)$$

and

$$f_c = \rho \omega^2 r \quad (9)$$

where g is the gravitational acceleration and r the distance from the axis of rotation. Then

$$\zeta = \tan^{-1} \frac{g}{\omega^2 r} \quad (10)$$

In commercial units the linear speed ωR of the rotor tip is 40-50 ft per second, while the radius R may be up to 2 feet. This gives a force angle of approximately 2°, indicating that the fluid will be centrifuged to the wall before it descends any appreciable distance. Thus only the paths along the wall and along the fillet are important in the analysis.

The Reynolds number is of the form $Re = \frac{Du\rho}{\mu}$ where u is the velocity of the fluid and D a characteristic linear dimension approximated by the ratio of the fluid cross-section to the wetted perimeter.

$$D = \frac{\pi R^2 - \pi (R - \delta)^2}{2\pi R} = \left(1 - \frac{\delta}{2R}\right) \quad (11)$$

For units with small clearance $\frac{\delta}{R} \ll 1$ and hence $D = \delta$ thus

$$Re = \frac{\delta u \rho}{\mu} \quad (12)$$

With small clearance and high rotor speed one might expect laminar flow in the space between the rotor tip and the shell, and turbulence in the fillet.

Downflow along the tube wall can be computed from the Navier-Stokes equation, which reduces in this case to

$$\frac{d^2\dot{z}}{dr^2} = \frac{-\rho g}{\mu} \tag{13}$$

where $\dot{z}$ is the downward velocity. The boundary conditions are that $\dot{z} = 0$ at the tube wall where $r = R$ and $\frac{d\dot{z}}{dr} = 0$ at the free surface where $r = R - \delta$. This gives for the velocity

$$\dot{z} = \frac{\rho g}{2\mu}\left[(R-r)^2 - 2\delta(R-r)\right] \tag{14}$$

and for the downflow V_w in volume per unit time

$$V_w = \int_R^{R-\delta} 2\pi r \dot{z}\, dr = \frac{2\pi R \rho g \delta^3}{3\mu}\left(1 - \frac{5\delta}{8R}\right) \tag{15}$$

Eq. (15) is the familiar result for film drainage in a wetted-wall column.

Computation of downflow in the fillet is more difficult. When the fillet is small, it is reasonable to treat the downflow as a problem in laminar flow, even though turbulent circulatory flow is to be expected in the horizontal plane. The Navier-Stokes equation for the vertical component of velocity is

$$\frac{\partial^2\dot{z}}{\partial x^2} + \frac{\partial^2\dot{z}}{\partial y^2} = \frac{-\rho g}{\mu} \tag{16}$$

With the origin at the intersection of the shell and the extension of the leading edge of the blade, the boundary conditions are $\dot{z} = 0$ at the shell wall where $x = 0$, $\dot{z} = 0$ at the blade wall where $y = 0$, and grad $\dot{z} = 0$ at the free surface.

If the free surface is assumed to be planar at 45° to the blade, the

problem can be related to the flow in a tube of square cross-section. Stokes[4] derived a theorem which states, in effect, that the flow in an open channel having a planar free surface is identical in velocity with that in a closed channel whose walls consist of the walls of the channel and their mirror images projected through the free liquid surface. Boussinesq[5] showed that the volume rate of flow of fluid through a vertical tube of cross-section X is

$$V = K_1 \frac{\rho g}{\mu} X^2 \tag{17}$$

where K_1 is a constant characteristic of the tube shape. For a square cross-section, $K_1 = 0.1405$, and hence for a fillet whose shape is approximately triangular and whose flow and cross-section are each half that in the corresponding square closed channel, $K_1 = 0.0703$. Eq. (17) would be valid for other fillet shapes, with slightly different values of K_1.

The volume of fluid in the processor at any instant is the holdup W, which consists of the fillet volume plus the wall film volume

$$W = 2\pi RL\delta + nLX \tag{18}$$

where X is the average cross-section area of a fillet. The throughput is the sum of the downflow along the wall and in the n fillets.

$$V = V_w + n V_f = \frac{2\pi R \rho g \delta^3}{3\mu} + 0.07 \frac{n\rho g}{\mu} X^2 \tag{19}$$

Solving for X in Eq. (19), we obtain

$$X = 3.8 n^{-1/2} \left[\frac{\mu V}{\rho g} - \frac{2\pi R \delta^3}{3}\right]^{1/2} \tag{20}$$

which gives for the holdup

$$W = \int_0^L \left(2\pi R\delta + nX\right) dz = 2\pi R\delta L + 3.8\, n^{1/2} \int_0^L \left(\frac{\mu V}{\rho g} - \frac{2}{3}\pi R\delta^3\right) dz \quad (21)$$

Since the limiting inside thermal resistance is the film of liquid adhering to the wall, the heat transferred Q is

$$Q = hA\,\Delta t_w \quad (22)$$

where A is the area of the processor shell and Δt_w may be taken as the difference between the inside shell temperature and the vapor temperature or the bulk liquid temperature in the absence of vaporization. The heat transfer coefficient h in the shell depends upon the thermal conductivity k of the fluid and the film thickness δ at the wall.

$$h = k/\delta \quad (23)$$

This relationship will give extremely conservative values of h if any appreciable convection takes place. However, convection should not be too important in the thin film with viscous fluids of the type generally handled in mechanically-aided thermal processors.

The analysis thus far has been limited to the isothermal flow of a non-vaporizing Newtonian fluid through the processor. In practice, the wall is heated and the fluid may be volatile and non-Newtonian. As the result of incremental process changes, the viscosity, density and thermal conductivity of the fluid may change appreciably as it descends. However, the approach used in the preliminary analysis will prove useful in analyzing the more complex situations, since it gives the fluid distribution as a

function of the properties of the fluid at any point as it travels through the thermal processor.

There are, in addition, other elements which are modifications of the rotor blade analyzed here. One of these employs both a tapered shell and tapered rotor. By changing the position of the rotor relative to the shell the clearance may also be altered. If the diameter of the taper increases toward the fluid outlet, the centrifugal force increases and the flow components along the taper are similarly increased beyond the values obtainable by gravity alone. This is a simple modification of the foregoing equations applicable to both vertical and horizontal shaft axes.

In any of these rigid blade rotors the blades may be notched at the edges to facilitate uniform distribution of the descending fluid. In another type of element the blade is free to move out radially by centrifugal force. The blade may also be notched. If the blade has no convergence between its edge and the free fluid surface, the analysis represents a simple adjustment of forces to include the centrifugal force of the blade. Where convergence occurs between the blade edge and the free fluid surface the analysis will follow the form outlined in the next section.

ROTORS WITH HYDRODYNAMIC BLADES

In several successful mechanically-aided thermal processors, the necessary close clearance between blade and shell is obtained by accurately machining the rotor and shell. The precision of the clearance also varies with thermal expansion when heat is supplied the jacket and as a

result of successive shutdowns and startups. To eliminate the need for close tolerances and precise balancing, it would be desirable to develop a processor in which the clearance is dynamically maintained. With properly designed rotor tips, this objective can be achieved through a balance between centrifugal and hydrodynamic forces.

Consider a trailer blade such as that shown in Fig. 3, with tip of mass m free to pivot about the bearing at point P. A leader blade can be treated in the same manner with appropriate changes of signs. When the blade arm rotates at an angular velocity ω , the blade is thrown radially outward by a centrifugal force F_c, where

$$F_c = m\omega^2 r = \frac{mu^2}{R} \tag{24}$$

The action of the fluid under shear in the truncated wedge-shaped clearance between the blade tip and the stationary wall produces a thrust against the blade surface. This situation is hydrodynamically identical with the slider bearing lubrication problem, which is treated in standard reference works on lubrication[6,7]. Only the applicable results are employed below.

Let w be the flat width of the blade face which is inclined at an angle Θ to the cylindrical wall, and let δ_o be the clearance between the mid-point of the flat blade width and the wall. The minimum clearance is

$$\delta = \delta_o(1-a) \tag{25}$$

where

$$a = \frac{w \tan\Theta}{2\,\delta_o} \tag{26}$$

The slider bearing force can be resolved into a <u>lift</u> component F_l which

is directed radially inward, and a drag component F_d , tangential to the direction of motion. For small values of Θ , these forces are

$$F_l = \frac{2w\mu L u}{\delta_o \tan\Theta} \cdot \psi(a) \tag{27}$$

and

$$F_d = \frac{2w\mu L u}{\delta_o} \cdot \psi(a) \tag{28}$$

where

$$\psi(a) = \frac{3}{2a} \ln \frac{1+a}{1-a} - 3 \tag{29}$$

and L may actually be comprised of a number of blades of shorter length. At dynamic equilibrium, the lift and centrifugal forces balance

$$\frac{mu^2}{R} = \frac{2w\mu L u}{\delta_o \tan\Theta} \cdot \psi(a) \tag{30}$$

Eq. (30) can be rearranged to give the governing equation relating clearance to blade velocity

$$\frac{mu}{2\mu R L} = \frac{w\,\psi(a)}{\delta_o \tan\Theta} \tag{31}$$

The power P_d required to overcome the viscous drag at the blade tip is

$$P_d = nF_d\,u \tag{32}$$

Since $F_d = F_l \tan\Theta$ and $F_l = F_c$,

$$P_d = \frac{nmu^3}{R} \tan\Theta \tag{33}$$

The most interesting feature of the power consumption equation is the absence of the fluid viscosity. This result is obtained because the clearance will be greater, the more viscous the fluid. We must also consider the power required to accelerate the holdup. If it is assumed that a fraction s of the holdup is accelerated from rest to the velocity u once each revolution the power P_s can be computed from

$$P_s = \left(\frac{W\rho u^2}{2g}\right)\left(\frac{u}{2\pi R}\right)s = \frac{sW\rho u^3}{4\pi g R} \tag{34}$$

where s must be determined experimentally to establish the actual slippage. Frictional losses in seals, stuffing boxes and drives are not included as a part of this study.

ANALYSIS OF FIXED AND HYDRODYNAMIC BLADES INCLUDING HEAT AND MASS TRANSFER

Because of the vaporization of solvent from the feed, the concentration of the less volatile components will increase with flow down the processor. The increased concentration will usually increase the viscosity and tend to increase the size of the fillet. The loss of solvent, on the other hand, acts to decrease the size of the fillet. Under certain operating conditions, for example with substantially pure solvent feed, the fillet will decrease along the unit. With a feed whose viscosity is greatly dependent on concentration, the fillet may increase.

If the heat flux in the processor is taken as Q/A, and the latent heat of the fluid is λ , then the volatilized volume dV in a length dz is

$$\lambda \rho \, dV = -2\pi K \left(\frac{Q}{A}\right) dz = -\frac{Q}{L} \, dz \tag{35}$$

Assuming the heat transferred per unit area to be uniform along the column, by integrating from the top of the column, where $z = 0$ and $V = V_o$, to a point at a distance z from the top, one obtains

$$V = V_o \left(1 - \alpha \frac{z}{L}\right) \tag{36}$$

where

$$\alpha = \frac{Q}{\lambda \rho V_o} \tag{37}$$

The dimensionless parameter α represents the fraction by volume of the feed which is vaporized in the processor. The concentration ϕ at height z can also be expressed in terms of the feed concentration ϕ_o and the parameter α.

$$\phi = \frac{\phi_o}{1 - \alpha z/L} \tag{38}$$

The fillet cross-section and the holdup are dependent on viscosity, which in turn depends upon concentration. Using, for example, Eq. (5) to represent the concentration dependence of viscosity with α', β' and u_o determined in advance for the particular fluid being processed, it is possible to compute the fillet cross-section at various values of z. First,

$$\mu = \mu_o \left(\frac{1 - \alpha'\phi}{1 - \beta'\phi}\right)^2 = \mu_o \left[\frac{1 - \dfrac{\alpha'\phi_o}{(1 - \alpha z/L)}}{1 - \dfrac{\beta'\phi_o}{(1 - \alpha z/L)}}\right]^2 \tag{39}$$

This value is then inserted in Eq. (20), with the term in δ^3 dropped as negligible, giving

$$X = 3.8 \left[\frac{1 - \dfrac{\alpha'\phi_o}{(1 - \alpha z/L)}}{1 - \dfrac{\beta'\phi_o}{(1 - \alpha z/L)}}\right] \left[\frac{\mu_o V_o}{n \rho g}\left(1 - \alpha z/L\right)\right]^{1/2} \tag{40}$$

To compute the holdup from Eq. (21), this value of X is used.

$$W = 2\pi R\delta L + 3.8\left[\frac{n\mu_0 V_0}{\rho g}\right]^{1/2}\int_0^L \left(\frac{1-\alpha z/L-\alpha'\phi_0}{1-\alpha z/L-\beta'\phi_0}\right)\left(1-\alpha z/L\right)^{1/2} dz \qquad (41)$$

This integral can be evaluated numerically with the aid of a digital computer, or with sufficient accuracy by means of Simpson's rule, using three values of z at 0, $L/2$ and L.

It becomes apparent in reviewing the foregoing equations that the same effect can be produced by employing a blade of large mass and low velocity or small mass and high velocity. There is a practical limitation to the range.

When processing corrosive fluids it is often desirable to fabricate the rotor and shell of metals which gall such as the stainless steels and monel, or of non-galling materials which are subject to wear. If the descending fluid is to serve as a lubricant, it is imperative that the flutter of the blades be suppressed. This is to prevent the blade from touching the shell at high velocities because of irregularities in the shell diameter or because of the fluid distribution which may deviate greatly from the ideal used in the derivations. On the other hand, it is equally imperative that the mass of the blade be restricted so that it does not strike the shell wall during the initial acceleration of the rotor on start-up and before steady-state operation is attained.

There are many possible modifications of the hydrodynamic blade discussed herein. In one type of blade it is possible to have a cut-out in

the blade so as to eliminate the fillet. If the slope of the cut-out surface is opposite that of the blade edge, it will oppose the lift force. If the cut-out surface is parallel to tangents to the shell inside diameter, it will only represent a drag. All of these influences can be included in the computation by a simple modification of the general force equations already presented.

Another type of modification opposes the action of the hydrodynamic blade. By completing the edge of the blade so that it possesses an apex in the direction of motion, it is possible to shear materials from the shell, thereby accomplishing a scraper effect. This is of particular value when dealing with liquids having properties which are not amenable to the hydrodynamic balance of forces.

This frequently occurs when it is desired to evaporate a solid solute to dryness in a single stage, an operation not uncommon in mechanically-aided thermal processors. In this event the blades in the principal evaporating section may be constructed of hydrodynamic blade increments, and the blades in the liquisolids region may be constructed of scraper or shear increments. In either event it may be necessary to face the rotor edge with a passive or plastic material if the rotor and shell materials gall or wear. The use of trailer or leader blades affects the signs used in summing the forces.

ILLUSTRATIVE EXAMPLE

It is desired to select a thermal processor to remove 2500 lbs/hr of

water from an aqueous Newtonian feed of 7500 lbs/hr having a viscosity of 100 centipoises (242 lb/ft x hr) and density of 62.4 lbs/ft^3. The jacket heating medium will be steam at 30 psig. On the basis of the jets available the internal working pressure will be 27 in. Hg vacuum. Referring to the commercial literature we find that a standard processor is available with an inside shell diameter of 26 inches and length 19.4 feet. For this illustration, assume the temperature of the shell inside wall is the same as the temperature of the steam.

$$\Delta t_w = 274 - 115 = 159\,°F$$

$$A = \pi \left(\frac{26}{24}\right) (19.4) = 66\ ft^2$$

$$Q = 2500 \times 1030 = 2,580,000\ Btu/hr$$

The required heat transfer coefficient is

$$h = \frac{Q}{A \Delta t_w} = \frac{2,580,000}{(66)\ (159)} = 246\ Btu/hr\ ft$$

Taking the thermal conductivity of the fluid as that of water, $k = 0.36$ Btu/hr x ft^2 x °F/ft, the required film thickness is

$$\delta = \frac{k}{h} = \frac{0.36}{246} = 0.00146\ ft = 0.017\ in.$$

This clearance, about 1/64 in., is too small to maintain with a fixed blade. For a hydrodynamic blade of width $w = 0.5$ in., angle $\Theta = 5°$, and mass $m = 5$ lbs., we have

$$\delta_0 = \delta + \frac{w}{2} \tan \Theta = 0.017 + \left(\frac{0.50}{2}\right)(0.0875) = 0.039\ in.$$

$$a = \frac{w \tan \Theta}{2 \delta_0} = \left(\frac{0.50}{2}\right) \frac{(0.0875)}{(0.039)} = 0.56 \qquad \text{[See Eq.(26)]}$$

$$\Psi(a) = \frac{3}{2a} \ln \frac{1+a}{1-a} - 3$$

$$= \frac{3}{2(0.56)} \ln \frac{1.56}{0.44} - 3 = 0.38 \qquad \text{[See Eq.(29)]}$$

Compute the dimensionless ratio $mu/2\mu RL$

$$\frac{mu}{2\mu RL} = \frac{w\,\Psi(a)}{\delta_o \tan\Theta} = \frac{(0.5)(0.38)}{(0.039)(0.0875)} = 55 \qquad \text{[See Eq.(31)]}$$

From this, the blade tip velocity is

$$u = \frac{(2)\,(55)\,(242)\,(13)(19.4)}{(5)\,(12)} = 1.1 \times 10^5 \text{ ft/hr} = 31 \text{ ft/sec.}$$

Taking $n = 4$ blades, the power consumption to overcome drag is

$$P_d = \frac{n\,m\,u^3}{R} \tan\Theta = \frac{(4)(5)(31)^3(0.0875)}{\frac{(13)}{(12)}\,(32)} = 1500\,\frac{\text{ft-lbs}}{\text{sec}} = 2.7 \text{ H.P.} \qquad \text{[See Eq.(33)]}$$

This is the power needed to overcome the viscous drag at the blade tips. Additional power will be needed to accelerate the fluid. The fillet cross-section is

$$X = 3.8\,n^{-1/2}\left(\frac{\mu V}{\rho g} - \frac{2\pi R\delta^3}{3}\right)^{1/2} = 3.8\left(\frac{\mu V}{n\rho g}\right)^{1/2} \qquad \text{[See Eq.(20)]}$$

since the wall film is only 0.00144 ft thick.

$$X = 3.8\left[\frac{(242)(7500)}{(4)(62.4)(4.18 \times 10^8)}\right]^{1/2} = 0.002 \text{ ft}^2$$

This is the fillet cross-section at the input. With evaporation the cross-section will vary along the unit. With viscosity-concentration information the variation can be computed from Eq. (40). For simplicity here, however,

it will be assumed that the cross-section is uniform along the column. The holdup is thus

$$W = 2\pi R\delta L + nXL$$
$$= 2(3.14)\left(\frac{13}{12}\right)(0.00146)(19.4) + 4(19.4)(0.002)$$
$$= 0.35 \text{ ft}^3 \ [\times 62.4] = 22 \text{ lbs}$$

Of these 22 lbs, 12 lbs are on the wall and 10 lbs in the fillets.

The power required for acceleration when s = 1.0 is

$$P_s = \frac{s W \rho u^3}{4\pi g R} = \frac{(1.0)(0.35)(62.4)(1.1\times10^5)^3}{4(3.14)(4.18\times10^8)(13/12)} \quad \text{[See Eq. (34)]}$$
$$= 5{,}100{,}000 \text{ ft lb/hr} \left[\times \frac{1}{550\times60\times60}\right] = 2.6 \text{ H.P.}$$

Actually the power required for acceleration will be considerably less since s will be fractional.

ACKNOWLEDGMENT

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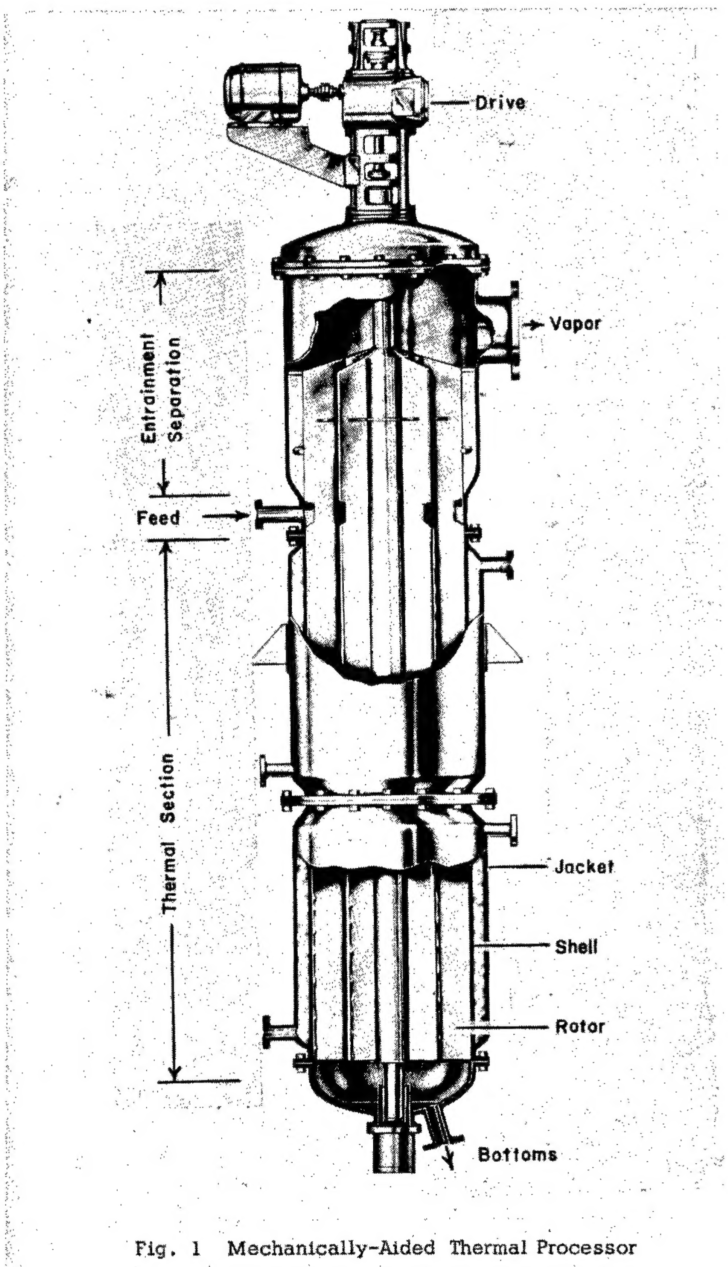

Fig. 1 Mechanically-Aided Thermal Processor

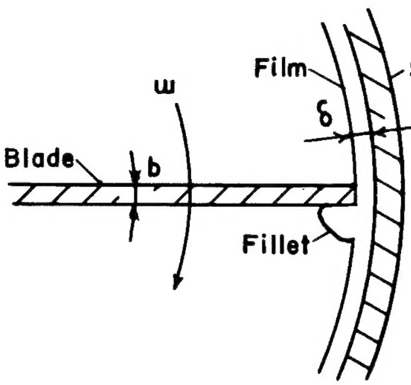

Fig. 2 Fixed Blade

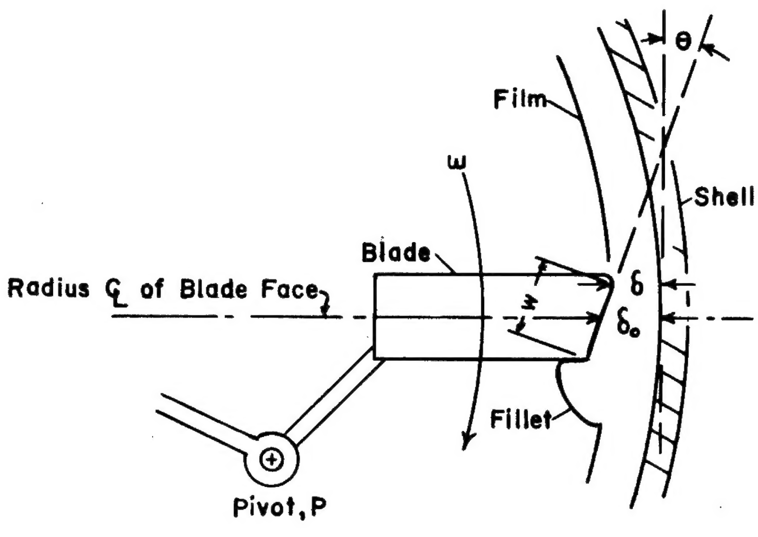

Fig. 3 Hydrodynamic Blade